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1989 J. Phys.: Condens. Matter 1 9413

(http://iopscience.iop.org/0953-8984/1/47/011)

J. Phys.: Condens. Matter 1 (1989) 9413–9420. Printed in the UK

The conductivity and dimensionality crossover of the disordered layered system in a magnetic field

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Received 16 February 1989, in final form 16 May 1989

Abstract. In this paper an interplay between the effects of disorder, interlayer coupling and magnetic field is studied in the disordered layered system. We present a crossover effect of dimensionality of the system, which depends on both the magnetic field $\boldsymbol{H}$ and the interlayer coupling t. The corrections of conductivity, $\Delta\sigma_j(H, T)$, corresponding to the maximal crossed diagrams are calculated. It is shown that the conductivities in different directions are anisotropic.

1. Introduction

In the study of the metal–insulator transition in Anderson localisation [1], it is shown from perturbation theory [2, 3] that for weak disorder, i.e for $(K_F l)^{-1} \ll 1$, where K_F is the Fermi velocity and l the mean-free path, if time reversal symmetry is satisfied, the quantum interference effects will lead to the occurrence of Anderson localisation. However, an external magnetic field breaks time reversal symmetry, the interference will be destroyed, resulting in the suppression of the effect of localisation [4]. In two dimensions one could consider the weak-field case $\omega_c\tau \ll 1$, ω_c being the cyclotron frequency and τ the mean scattering time, the resulting correction of the conductivity $\delta\sigma_H \sim \ln H$ [5], which means that there exists the so-called negative magnetoresistance effect. This effect was experimentally observed on Si inversion layers [6, 7].

In this paper a model of a disordered layered system in an external magnetic field is studied. In recent years there has been a resurgence of interest in the properties of the disordered multilayered materials both theoretically and experimentally [8–10]. Our model simulating this structure is composed of 2D like disordered layers and between them insulating materials intercalated. By varying the thickness of insulating materials, the interlayer coupling t whose value characterises the strength of interlayer coupling, could be changed, and then the crossover behaviour from 2D to 3D may occur in the system. We assume that a magnetic field $\boldsymbol{H}$ is applied along the z axis perpendicular to the layer and put

$$\boldsymbol{A} = (0, Hx, 0).$$

In the presence of weak magnetic field satisfying $\omega_c\tau \ll 1$, we studied the interplay

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between disorder, interlayer coupling and magnetic field. We have obtained the crossover behaviour of the conductivities, depending on the applied magnetic field, from 2D to 3D. It is pointed out that for fields such that $L_H = (\hbar c/eH)^{1/2} > L_t$, $L_t = V_F/t$ being the characteristic length in the layered system, the system has 3D characteristics, otherwise the system behaves like a 2D one. In the presence of a magnetic field the corrections of the conductivities, $\Delta\sigma_j(H, T)$ $(j = \|, \perp)$ corresponding to the maximally crossed diagrams have been calculated. It is found that in the limit $t \to 0$, we obtain $\Delta\sigma_{\|}(H, T) \sim \ln H$, and in 3D, $\Delta\sigma_j(H, T) \sim H^{1/2}$. We also note that $\Delta\sigma_j(H, T)$ in different directions are anisotropic.

In the following, we are confined to discussing non-interacting electron systems. We assume the impurity potential to be effective only within a given layer. Then the potentials located on different layers are statistically independent.

2. The vertex functions in the magnetic field

In general, in a magnetic field an electron undergoes cyclotron motion in a limited space even without disorder, and it might seem that the localisation is enhanced. This argument might be correct if the mean scattering time τ is long enough for an electron to perform cyclotron motion, i.e. if $\omega_c\tau \gg 1$. On the other hand, if magnetic field is very weak, i.e. if $\omega_c\tau \ll 1$, this argument will not be true. It is found that the conductivities are sensitively affected by the applied magnetic field. In the case of a weak magnetic field the effects of energy level quantisation do not show up. Nevertheless, the modification of the phase of the wavefunctions should not be neglected, and we have

$$\Psi(\boldsymbol{r}) \to \exp\left[(\mathrm{i}e/\hbar c)\int_{r_0}^{r} \boldsymbol{A}(\boldsymbol{r}')\cdot \mathrm{d}\boldsymbol{r}'\right]\Psi(\boldsymbol{r}) \tag{1}$$

where $\boldsymbol{H} = \nabla \times \boldsymbol{A}$. If an electron propagates along a closed path, the phase shift after the motion is given by

$$(e/\hbar c)\oint \boldsymbol{A}(\boldsymbol{r}')\cdot \mathrm{d}\boldsymbol{r}' = e\Phi/\hbar c \tag{2}$$

where Φ is the magnetic flux penetrating inside the closed path. The phase shift of an electron propagating along the same path in the opposite direction is the same as (2) with the opposite sign. If the linear dimension of the closed path is L, then Φ is of the order HL^2. Thus it is expected that the phase difference of the order HL^2 appears between two waves propagating on the reversed course. If the difference becomes of the order of unity, the quantum interference resulting in the localisation will be destroyed. For this case a characteristic length is estimated as

$$L_H \sim (\hbar c/eH)^{1/2}. \tag{3}$$

Since $L_H \gg l$, in the following the cutoff parameters $1/l$ for the case without magnetic field [4] will be replaced by $1/L_H$.

If $G(\boldsymbol{r}, \boldsymbol{r}', \varepsilon)$ is the Green function in the absence of an external magnetic field then in the presence of a magnetic field it is given by

$$G_H(\boldsymbol{r}, \boldsymbol{r}', \varepsilon) = G(\boldsymbol{r}, \boldsymbol{r}', \varepsilon) \exp\left((\mathrm{i}e/\hbar c) \int_{\boldsymbol{r}}^{\boldsymbol{r}'} \boldsymbol{A}(s) \cdot \mathrm{d}s\right). \tag{4}$$

It suffices to study the vertex functions $\Gamma_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$ which contribute to the conductivities with Kubo formula. In real space the Dyson equation with respect to Γ_H is given by

$$\Gamma_H(\boldsymbol{r}, \boldsymbol{r}', \omega) = n_i v^2 \delta(\boldsymbol{r} - \boldsymbol{r}') + n_i v^2 \sum_{\boldsymbol{r}_1} \Pi_H(\boldsymbol{r}, \boldsymbol{r}_1, \omega) \Gamma_H(\boldsymbol{r}_1, \boldsymbol{r}', \omega) \tag{5}$$

where

$$\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega) = G_H^{\mathrm{R}}(\boldsymbol{r}, \boldsymbol{r}' \omega) G_H^{\mathrm{A}}(\boldsymbol{r}, \boldsymbol{r}', \varepsilon - \omega) \tag{6}$$

n_i is the density of impurities on layers, v^2 the mean-square impurity potential and $G_H^{\mathrm{R(A)}}(\boldsymbol{r}, \boldsymbol{r}', \varepsilon)$ represent the retarded (advanced) Green function in the presence of the field.

Putting (4) into (6) we can obtain

$$\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega) = \Pi(\boldsymbol{r} \cdot \boldsymbol{r}', \omega) \exp\left((2\mathrm{i}e/\hbar c) \int_{\boldsymbol{r}}^{\boldsymbol{r}'} \boldsymbol{A}(s) \cdot \mathrm{d}s\right) \tag{7}$$

where

$$\Pi(\boldsymbol{r} \cdot \boldsymbol{r}', \omega) = G^{\mathrm{R}}(\boldsymbol{r}, \boldsymbol{r}', \varepsilon) G^{\mathrm{A}}(\boldsymbol{r}, \boldsymbol{r}', \varepsilon - \omega). \tag{8}$$

We know from (5) that in order to evaluate $\Gamma_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$, it is necessary to find out the solution of $\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$. In the following we extend the methods of Altshuler *et al* [5] to the study of the anisotropic layered system. For this purpose we consider $\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$ as an operator with its eigenfunctions $\psi_\eta(\boldsymbol{r})$ such that

$$\int \Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega) \psi_\eta(\boldsymbol{r}') \, \mathrm{d}\boldsymbol{r}' = \lambda(\eta) \psi_\eta(\boldsymbol{r}) \tag{9}$$

namely

$$\int \Pi(\boldsymbol{r}, \boldsymbol{r}' \omega) \exp\left((2\mathrm{i}e/\hbar c) \int_{\boldsymbol{r}}^{\boldsymbol{r}'} \boldsymbol{A}(s) \cdot \mathrm{d}s\right) \psi_\eta(\boldsymbol{r}') \, \mathrm{d}\boldsymbol{r}' = \lambda(\eta) \psi_\eta(\boldsymbol{r}). \tag{10}$$

Using the identical equation [11] we obtain

$$\exp\left[(2\mathrm{i}e/\hbar c) \int_{\boldsymbol{r}}^{\boldsymbol{r}'} \boldsymbol{A}(s) \cdot \mathrm{d}s\right] \psi_\eta(\boldsymbol{r}') = \exp[-\mathrm{i}(\boldsymbol{r} - \boldsymbol{r}')(\nabla/\mathrm{i} + 2e\boldsymbol{A}/\hbar c)] \psi_\eta(\boldsymbol{r}). \tag{11}$$

At the present, we put $\boldsymbol{A} = (0, Hx, 0)$, and define $\boldsymbol{r} = (r_\parallel, r_\perp)$ and $\nabla = (\nabla_\parallel, \nabla_\perp)$, with $\boldsymbol{r}_\parallel = (x, y)$, $r_\perp = z$, $\nabla_\parallel = (\partial/\partial x, \partial/\partial y)$ and $|\nabla_\perp| = \partial/\partial z$. Expanding $\boldsymbol{A}(\boldsymbol{r})$ and $\psi_\eta(\boldsymbol{r})$ about $\boldsymbol{r}_\parallel$ and $\boldsymbol{r}_\perp$ up to second order, we have

$$\exp\left[(2\mathrm{i}e/\hbar c) \int_{\boldsymbol{r}}^{\boldsymbol{r}'} \boldsymbol{A}(s) \cdot \mathrm{d}s\right] \psi_\eta(\boldsymbol{r}') \simeq [1 - \tfrac{1}{2}(\boldsymbol{r}_\parallel - \boldsymbol{r}_\parallel')^2 (\nabla_\parallel/\mathrm{i} + 2e\boldsymbol{A}/\hbar c)^2 + \tfrac{1}{2}(\boldsymbol{r}_\perp - \boldsymbol{r}_\perp')^2 \nabla_\perp^2] \psi_\eta(\boldsymbol{r}). \tag{12}$$

On the other hand, in the absence of the magnetic field we may define the Fourier transform of $\Pi(\boldsymbol{r}, \boldsymbol{r}', \omega)$ as

$$\Pi(\boldsymbol{r}, \boldsymbol{r}', \omega) = (1/2\pi)^3 \int \mathrm{d}\boldsymbol{q}_{\parallel} \int \mathrm{d}\boldsymbol{q}_{\perp} \exp[\mathrm{i}\boldsymbol{q}_{\parallel} \cdot (\boldsymbol{r}_{\parallel} - \boldsymbol{r}'_{\parallel}) + \mathrm{i}\boldsymbol{q}_{\perp} \cdot (\boldsymbol{r}_{\perp} - \boldsymbol{r}'_{\perp})]\Pi(\boldsymbol{q}_{\parallel}, \boldsymbol{q}_{\perp}, \omega) \tag{13}$$

with $\boldsymbol{q} = (q_{\parallel}, q_{\perp})$. This yields

$$\Pi(q = 0, \omega) = \int \mathrm{d}\boldsymbol{r}' \, \Pi(\boldsymbol{r}, \boldsymbol{r}', \omega) \tag{14}$$

and

$$\begin{aligned} \int \Pi(\boldsymbol{r}, \boldsymbol{r}', \omega)(\boldsymbol{r}_{\parallel} - \boldsymbol{r}'_{\parallel})^2 \, \mathrm{d}\boldsymbol{r}' &= -\frac{\partial^2}{\partial q_{\parallel}^2} \Pi(\boldsymbol{q}, \omega)\bigg|_{q=0} \\ \int \Pi(\boldsymbol{r}, \boldsymbol{r}', \omega)(\boldsymbol{r}_{\perp} - \boldsymbol{r}'_{\perp})^2 \, \mathrm{d}\boldsymbol{r}' &= -\frac{\partial^2}{\partial q_{\perp}^2} \Pi(\boldsymbol{q}, \omega)\bigg|_{q=0}. \end{aligned} \tag{15}$$

Putting (12), (14) and (15) into (10), we obtain

$$\left(\Pi(q = 0, \omega) + \tfrac{1}{2} \frac{\partial^2 \Pi(\boldsymbol{q}, \omega)}{\partial q_{\parallel}^2}\bigg|_{q=0} (\nabla_{\parallel}/\mathrm{i} + 2e\boldsymbol{A}/\hbar c)^2 - \tfrac{1}{2} \frac{\partial^2 \Pi(\boldsymbol{q}, \omega)}{\partial q_{\perp}^2}\bigg|_{q=0} \nabla_{\perp}^2\right) \psi_{\eta}(\boldsymbol{r}) = \lambda(\eta)\psi_{\eta}(\boldsymbol{r}). \tag{16}$$

Defining

$$\begin{aligned} &\lambda'(\eta) = \lambda(\eta) - \Pi(q = 0, \omega) \\ &1/m_{\parallel} = \frac{\partial^2 \Pi(\boldsymbol{q}, \omega)}{\partial q_{\parallel}^2}\bigg|_{q=0} \qquad 1/m_{\perp} = \frac{\partial^2 \Pi(\boldsymbol{q}, \omega)}{\partial q_{\perp}^2}\bigg|_{q=0}. \end{aligned} \tag{17}$$

We could rewrite (16) as

$$[(1/2m_{\parallel})(\nabla_{\parallel}/i + 2e\boldsymbol{A}/\hbar c)^2 - (1/2m_{\perp})\nabla_{\perp}^2]\psi_{\eta}(\boldsymbol{r}) = \lambda'(\eta)\psi_{\eta}(\boldsymbol{r}). \tag{18}$$

Equation (18) demonstrates that the eigenfunctions of $\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$ are identical with the wavefunctions of a doubly charged particle of anisotropic mass in different directions in a magnetic field along the z axis. The results of this eigenvalue problem are well known, and from (17) we have (taking $c = 1$)

$$\lambda_n = \Pi(q = 0, \omega) + (n + \tfrac{1}{2})2e\hbar H/m_{\parallel} + q_{\perp}^2/2m_{\perp}. \tag{19}$$

In the absence of the magnetic field we have determined $\Pi(\boldsymbol{q}, \omega)$ [10] as follows

$$\Pi(\boldsymbol{q}, \omega) = (2\pi N(E_{\mathrm{F}})\tau/\hbar)[1 + \mathrm{i}\omega\tau - D_{\parallel}^0 q_{\parallel}^2 \tau - D_{\perp}^0 \tau(1 - \cos q_{\perp} a)/a^2] \tag{20}$$

with $N(E_{\mathrm{F}})$ the density of states at the Fermi level and $D_{\parallel}^0$ and $D_{\perp}^0$ the bare diffusion coefficients parallel and perpendicular to the layer. Here $D_\parallel^0 = V_F^2\tau/2$, and $D_\perp^0 = t^2a^2\tau$, a being the interlayer spacing and t the interlayer coupling. We obtain

$$\lambda_n = (2\pi N(E_F)\tau/\hbar)[1 + i\omega\tau - (n + \tfrac{1}{2})4eHD_\parallel^0\tau/\hbar - D_\perp^0 q_\perp^2 \tau]. \tag{21}$$

It could be seen from (21) that in the magnetic field the term $(\nabla_\parallel/i + 2e\boldsymbol{A}/\hbar)^2$ is quantised, that is

$$q_\parallel^2 \to (n + \tfrac{1}{2})4eH/\hbar. \tag{22}$$

We may obtain $\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$ by multiplying (9) by $\psi_\eta(\boldsymbol{r}'')$ and summing over all η. It is given by

$$\Pi_H(\boldsymbol{r}, \boldsymbol{r}', \omega) = (2\pi N(E_F)\tau/\hbar)(2H/\pi\hbar) \sum_n \psi_n(\boldsymbol{r})\psi_n^*(\boldsymbol{r}') \times [1 + i\omega\tau - (n + \tfrac{1}{2})4eHD_\parallel^0\tau/\hbar - D_\perp^0 q_\perp^2 \tau]. \tag{23}$$

Hence putting (23) into (5), we can obtain the vertex functions in the coordinate representation in the form

$$\Gamma_H(\boldsymbol{r}, \boldsymbol{r}', \omega) = (1/2\pi N(E_F)\tau^2)(2H/\pi\hbar) \sum_n \psi_n(\boldsymbol{r})\psi_n^*(\boldsymbol{r}') \times [(n + \tfrac{1}{2})4eHD_\parallel^0/\hbar + q_\perp^2 D_\perp^0 - i\omega]^{-1}. \tag{24}$$

We expand $\Gamma_H(\boldsymbol{r}, \boldsymbol{r}', \omega)$ about $\psi_\eta(\boldsymbol{r})$, obtaining:

$$\Gamma_H(\boldsymbol{q}_\parallel, \boldsymbol{q}_\perp, \omega) = (1/2\pi N(E_F)\tau^2)1/[(n+\tfrac{1}{2})4eHD_\parallel^0/\hbar + q_\perp^2 D_\perp^0 - i\omega]. \tag{25}$$

It is easy to see from (25) that application of a magnetic field introduces a cut-off of the pole appearing in $\Gamma(\boldsymbol{q}, \omega)$ for the case with $\boldsymbol{H} = 0$ [4]. This will result in the suppression of the effect of localisation.

3. The corrections of conductivities and crossover of dimensionality in the layered system

The effect of the weak magnetic field on the Green function can only be taken into account as a phase shift. Thus the Kubo formalism [5] can be used to calculate the corrections of conductivities for the layered system in the magnetic field. The corrections of conductivities for the maximal cross diagrams can be obtained by using (25). We have

$$\delta\sigma_\parallel(\omega, H) = -\left(\frac{2e^2}{\pi\hbar}\right)\left(\frac{eH}{\pi\hbar}\right)\int_{-\pi/a}^{\pi/a} \frac{dq_\perp}{2\pi} \times \sum_{n=0}^{\hbar/4eH\tau D_\parallel^0} D_\parallel^0/[(n + \tfrac{1}{2})4eHD_\parallel^0/\hbar + D_\perp^0 q_\perp^2 - i\omega] \tag{26a}$$

$$\delta\sigma_\perp(\omega, H) = -\left(\frac{2e^2}{\pi\hbar}\right)\left(\frac{eH}{\pi\hbar}\right)\int_{-\pi/a}^{\pi/a} \frac{dq_\perp}{2\pi} \times \sum_{n=0}^{\hbar/4eH\tau D_\parallel^0} D_\perp^0/[(n+\tfrac{1}{2})4eHD_\parallel^0/\hbar + D_\perp^0 q_\perp^2 - i\omega] \tag{26b}$$

where $\delta\sigma_{\parallel}$ and $\delta\sigma_{\perp}$ are the corrections of conductivities parallel and perpendicular to the layer. The integration over $q_{\perp}$ in (26) should be performed over the whole Brillouin zone $-\pi/a < q_{\perp} < \pi/a$. However, we limit the sum to $n < \hbar/4eH\tau D_{\parallel}^{0}$. It is found from (26) that the pole in the situation with $\boldsymbol{H} = 0$ has been removed, resulting in the effect of delocalisation in the system. It follows from (26) that

$$\delta\sigma_{\parallel}(\omega, H)/\delta\sigma_{\perp}(\omega, H) = D_{\parallel}^{0}/D_{\perp}^{0}. \tag{27}$$

Therefore, we only discuss the situations of $\delta\sigma_{\parallel}(\omega, H)$ in the following. The temperature dependences of $\delta\sigma_{\parallel}(\omega, H)$ can be obtained by replacing $(-\mathrm{i}\omega)$ in (26) by $1/\tau_{\mathrm{in}}$, where τ_{in} is the inelastic scattering time. Performing the integration over $q_{\perp}$ in (26), we have (for simplicity, taking $\hbar = 1$):

$$\delta\sigma_{\parallel}(T, H) = -(e^2/\pi^3)(D_{\parallel}^{0}/D_{\perp}^{0})^{1/2}(1/L_H) \times \sum_{n=0}^{\hbar/4eH\tau D_{\parallel}^{0}} \frac{\tan^{-1}[(\pi/2)(D_{\perp}^{0}/D_{\parallel}^{0})^{1/2} L_H a/(n+\frac{1}{2}+\delta)^{1/2}]}{(n+\frac{1}{2}+\delta)^{1/2}} \tag{28}$$

with $\delta = 1/4eH\tau_{\mathrm{in}}D_{\parallel}^{0}$, and $L_H = (1/eH)^{1/2}$. We finally obtain the forms of the quantum corrections of conductivities in the layered system. It is evident from (28) that the conductivities are anisotropic. We discuss the influence of the interlayer coupling t on the conductivities, and if $L_H < L_t = V_{\mathrm{F}}/t$, then from (28) we obtain

$$\delta\sigma_{\parallel}(T, H) \simeq \left(\frac{e^2}{2\pi^2 a}\right) \sum_{n=0}^{1/4eH\tau D_{\parallel}^{0}} \frac{1}{(n+\frac{1}{2}+\delta)}. \tag{29}$$

Thus $\delta\sigma_{\parallel}(T, H)$ is almost independent of the coupling t. This means that the layer is effectively uncoupled from other such adjacent layers. Performing the summation over n, the following relation holds true:

$$\delta\sigma_{\parallel}(T, H) = -(e^2/2\pi^2 a)[\ln(1/4eH\tau D_{\parallel}^{0}) - \psi(\tfrac{1}{2} + 1/4eH\tau_{\mathrm{in}}D_{\parallel}^{0})] \tag{30}$$

where ψ is the digamma function, and use has been made of the limits $k_{\mathrm{F}}l \gg 1$, and $\omega_c\tau \ll 1$. If temperature is higher, such that $L_H \gg L_{\mathrm{T}}$, L_{T} being the Thouless length with $L_{\mathrm{T}} = (D_{\parallel}^{0}\tau_{\mathrm{in}})^{1/2}$, then from (30) we obtain

$$\delta\sigma_{\parallel} = -(e^2/2\pi^2 a)\ln(\tau_{\mathrm{in}}/\tau). \tag{31}$$

It is seen explicitly from (31) that a logarithmic dependence on τ_{in} is revealed, consistent with that of two dimensions [5].

For $L_H \ll L_{\mathrm{T}}$, from (30) we have a logarithmic dependence on H by

$$\delta\sigma_{\parallel} \sim \ln(4eHD_{\parallel}^{0}\tau). \tag{32}$$

This predicts a negative magnetoresistance, as it should, resulting in the suppression of the localisation effect. Equation (32) has typical two-dimensional characteristics.

For all the above, it is shown that if the inequality $L_H < L_t$ is satisfied, the system behaves like a 2D one. Let us now give an interpretation of the results. Firstly, for a certain magnetic field H, if the interlayer coupling t is very small, such that $L_H < L_t$, the layered system is considered as stacks of 2D layers which are independent of each other. Thus the spatial dimensionality of the system is two. On the other hand, for a certain interlayer coupling t, if the above inequality also is needed, then a large magnetic field H will make the system have a crossover behaviour from 3D to 2D, even though in the absence of the field the coupling t is enough to make it possible to locate the system in 3D. This argument might be checked by experiments.

If $L_H > L_t$, we know from (28) that the system behaves like a 3D one. In the weak magnetic field, we confine ourselves to the case $L_H \gg l$, with $l = (D_\parallel^0 \tau)^{1/2}$. Hence taking the limit of the summation in (28) as infinite, we obtain for $L_H \gg a$,

$$\delta\sigma_\parallel(T, H) = -\left(\frac{e^2}{2\pi^2 L_H}\right)\left(\frac{D_\parallel^0}{D_\perp^0}\right)^{1/2} \sum_{n=0}^{\infty} \frac{1}{(n + \frac{1}{2} + \delta)^{1/2}}. \tag{33}$$

If the correction of conductivity in the limit of $H \to 0$, is denoted by $\delta\sigma_\parallel(T, 0)$, the magnetoconductivity is given by

$$\Delta\sigma_\parallel(T, H) = \delta\sigma_\parallel(T, H) - \delta\sigma_\parallel(T, 0). \tag{34}$$

At low temperature or strong magnetic field, with $L_H \ll L_T$, from (33) and (34) we obtain

$$\Delta\sigma_\parallel(T, H) \simeq (C_1 e^2/2\pi^2 L_H)(D_\parallel^0/D_\perp^0)^{1/2} \tag{35}$$

where $C_1 = 0.605$. Hence a power dependence on H is given. This is a typical charcteristic in 3D, and the negative magnetoresistance effect appears in the system. For the layered structure, it is easy to see from (35) that the correction of conductivities is anisotropic, as it should be.

If $L_H \gg L_T$, corresponding to high temperature or weak magnetic field, we have

$$\Delta\sigma_\parallel(T, H) \simeq (C_2 e^2/2\pi^2 L_H)(D_\parallel^0/D_\perp^0)^{1/2}\delta^{-3/2} \tag{36}$$

with $C_2 = 1/48$. Thus it gives the results with $\Delta\sigma_\parallel(T, H) \sim H^2$.

From the above we may conclude that for a certain magnetic field H, when $L_H > L_t$, the system is located in 3D for larger interlayer coupling t. On the other hand, if the inequality is satisfied for a certain coupling t, a weak magnetic field may make it possible to cross from 2D to 3D in the dimensionality of the system, even if the coupling t is so small that the system is located in 2D spatially in the absence of the magnetic field. This implies that delocalisation may appear in the system.

4. Discussion

It has been noted in [13] that the field dependence of magnetoresistance has been considered for a magnetic field applied parallel or perpendicular to the plane in two-dimensional systems with effects of Zeeman splitting and spin–orbit scattering, and that the characteristic anisotropy has arisen from geometries of both parallel and perpendicular fields rather than from the interlayer coupling. Furthermore, in their discussion, crossover effect of dimensionality has not been investigated. In contrast, our results presented here may provide some information about an interplay between the effects of disorder, interlayer coupling and magnetic field. It is proved that in the presence of a perpendicular magnetic field the crossover of dimensionality in the layered system depends not only on the applied magnetic fields, but also on the interlayer coupling t. It has been shown that the corrections of conductivities in different directions are anisotropic. The general feature of anisotropic layered structures has been reflected in the expressions of $\delta\sigma_j \sim (D_\parallel^0/D_\perp^0)^{1/2}$, i.e. $\delta\sigma_j$ depends on the interlayer coupling t.

In conclusion, we point out that the present results can be verified within the framework of scaling theory of localisation. A general scaling claim is made that the conductivity can be scaled with an effective length which may depend on L_t, L_H, L_T, etc. In a previous paper [10] we applied a single parameter scaling theory to the disordered layered system without a magnetic field. Let $g(L_\|, L_\perp)$ be a dimensionless conductance of a sample of the size $L_\|^2 \cdot L_\perp$. Firstly, we considered the transverse dimension $L_\perp$ to be of the order of the interlayer distance a, and the conductance g, depending on the parallel size $L_\|$, undergoing a scaling transformation starting from l (l is the mean-free path). For $L_\| < L_t$ the scaling equation for g should be the same as in the pure 2D case. For $L_\| \simeq L_t$ a crossover from 2D to 3D behaviour took place. If $L_\perp \sim L_\|$, and $L_\| > L_t$, we could scale both $L_\|$ and $L_\perp$ with the same factor. The scaling transformation followed the 3D scaling equation. In the presence of a magnetic field, a similar procedure could be made by adding L_H as an additional length. In fact our approach in the present paper is nothing but the realisation of that program within diagrammatical description.

Finally, the method given here needs to be essentially improved in order to consider the case in the presence of a strong magnetic field where the quantum Hall effect would appear.

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